

# Sample Documents

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**IL Sec Math Contest**  
(ISC)

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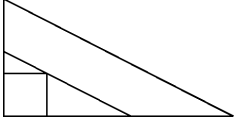
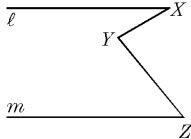
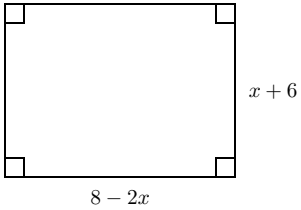
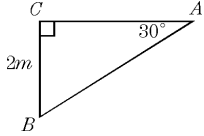
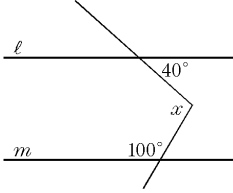
**April, 1999**  
**Wednesday**

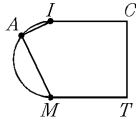
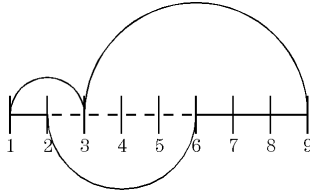
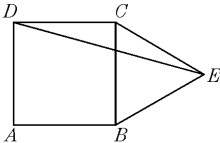
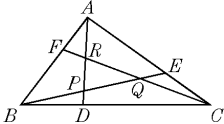
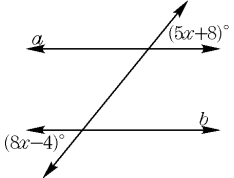
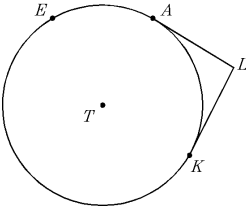
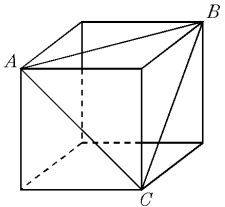
**Monday**

**Tuesday**

**Thursday**

**Friday**

<p>A triangle has sides 5, 12, 13. Two smaller triangles, similar to the original triangle, are drawn with one vertex at the larger triangle's incenter, as shown. Find the sum of the areas of the two smaller triangles. Give your answer as a reduced fraction.</p> 	<p>Find the equation of the line determined by the points of intersection of the circles with equations <math>x^2 - 2x + y^2 + 4y = -1</math> and <math>x^2 + y^2 + 2y = 3</math>. Give your answer in the form <math>Ax + By = C</math>, where <math>A</math> and <math>B</math> are relatively prime integers and <math>C &gt; 0</math>.</p>	<p>In the diagram shown, lines <math>\ell</math> and <math>m</math> are parallel. If <math>\angle X = 30^\circ</math> and <math>\angle Y = 80^\circ</math>, find the measure, in degrees, of <math>\angle Z</math>.</p> 	<p>1</p> <p>A regular hexagon <math>ABCDEF</math> is inscribed in a circle and has a circle circumscribed about it. The hexagon has a side of length 8 units. Find the ratio of the area of the inscribed circle to the circumscribed circle. Give your answer in the form <math>a : 1</math>, where <math>a</math> is a simplified fraction.</p>	<p>2</p> <p>A tetrahedron has 3 right angles at its vertex with each slanting edge of length 20. Find the volume.</p>
<p>5</p> <p>The average of the measures of seven interior angles of a convex polygon is <math>160^\circ</math>. What is the sum of the measures, in degrees, of the exterior angles of the remaining three interior angles.</p>	<p>6</p> <p>Find all possible areas of the rectangle drawn in the figure shown. Write your answer as an inequality with the area noted as <math>A</math> and <math>x \neq 0</math>.</p> 	<p>7</p> <p>A solid piece of wood in the shape of a regular square pyramid has a height of 24 and a base with a side of 18. A hole with radius 2 is drilled perpendicular to the base up through the solid with the center of the hole 3 units in from the side of the square along the line from the midpoint of the side to the center of the square. Find the volume, in cubic units, of the remaining solid.</p>	<p>8</p> <p>In triangle <math>ABC</math>, <math>AB = 8</math>, <math>BC = 6</math> and <math>AC = 12</math>. <math>D</math> is a point on <math>AC</math> so that <math>BD</math> bisects <math>\angle B</math>. Find the length of <math>AD</math>.</p>	<p>9</p> <p>Find the length of <math>\overline{AC}</math> in terms of <math>m</math>.</p> 
<p>12</p> <p>The second angle of a triangle contains 10 degrees more than the first angle while the third angle is 20 degrees larger than the first. What is the ratio of the measure of first angle to the measure of the third angle? Give your answer in the form <math>r : s</math>, where <math>r</math> and <math>s</math> are relatively prime, positive integers.</p>	<p>13</p> <p>A certain polyhedron has eleven faces. If <math>V</math> is the number of vertices and <math>E</math> is the number of edges, find <math>E^2 - 2EV + V^2</math>.</p>	<p>14</p> <p>Let <math>\triangle ABC</math> be a right triangle with right angle at <math>B</math>. Let <math>D</math> be the base of the altitude from <math>B</math> to side <math>AC</math>. If the length of <math>BD</math> is 6 and <math>\angle BAD = \angle DBC = 60^\circ</math>, find the perimeter of <math>\triangle ABC</math>. Give your answer correct to four significant digits.</p>	<p>15</p> <p>In the diagram shown, lines <math>\ell</math> and <math>m</math> are parallel. If the angles have the measures shown, what is the value of <math>x</math>, in degree measure?</p> 	<p>16</p> <p>A regular hexagon is inscribed in another regular hexagon by joining the adjacent midpoints of the outer hexagon's sides. The ratio if the area of the inner hexagon to the area of the outer hexagon can be given as <math>a : b</math>, where <math>a, b</math> are positive integers with no common factors. Give the ordered pair <math>(a, b)</math> as your answer.</p>

<p>19</p> <p>Let <math>m\angle A = 45^\circ</math>, <math>m\angle B = 30^\circ</math>, <math>m\angle C = 135^\circ</math>, <math>m\angle D = 90^\circ</math>, <math>m\angle E = 150^\circ</math>, <math>m\angle F = 120^\circ</math>, <math>m\angle G = 60^\circ</math>. Two angles are randomly chosen at the same time. What is the probability that they are supplementary? Give your answer as a reduced fraction.</p>	<p>20</p> <p>Arc <math>IAM</math> is a semicircle with <math>\overline{AI} = 3</math>, <math>\overline{AM} = 4</math>, <math>\overline{IC} = \overline{CT} = \overline{MT}</math>, <math>m\angle C</math> and <math>m\angle T = 90^\circ</math>. Find the area, in square units, of the pentagon <math>AICTM</math>.</p> 	<p>21</p> <p>A triangle has sides of 3, 25, and 26. Find the length of the altitude to the shortest side.</p>	<p>22</p> <p>A basketball has an outside radius of <math>28''</math> and an inside radius of <math>27.8''</math>. Find the volume, in cubic inches, of the material used to make the ball. Give your answer rounded to four significant digits.</p>	<p>23</p> <p>Three semi-circles, centered at 2, 4 and 6, respectively, are drawn along a number line as shown. Find the total perimeter of the figure.</p> 
<p>26</p> <p>Find the area, in square units, of an equilateral triangle inscribed in a circle of radius 12.</p>	<p>27</p> <p>In the diagram shown, <math>ABCD</math> is a square and <math>\triangle BCE</math> is equilateral. What is the measure, in degrees, of <math>\angle CED</math>?</p> 	<p>28</p> <p>The sum of the areas of two similar triangles is 144. The ratio of any two corresponding sides of the triangles is 3:1. Find the area of the larger triangle.</p>	<p>29</p> <p>Let <math>F, D, E</math> be points on the sides of <math>ABC</math> as indicated in the diagram shown. Suppose the area of <math>ABC</math> is 1 and</p> $\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB} = \frac{1}{2}.$ <p>Find the area of <math>\triangle PQR</math>.</p> 	<p>30</p> <p>Derive the equation of the line parallel to <math>3x - 5y = -7</math> which contains the point <math>(7, -2)</math>. Give your answer in the form <math>Ax + By = C</math>, where <math>A</math> and <math>B</math> are relatively prime integers and <math>C &gt; 0</math>.</p>
<p>Suppose line <math>a</math> is parallel to line <math>b</math> and a transversal cuts both lines giving angles as shown. What is the value of <math>x</math>?</p> 	<p>A cone and a sphere have equal volumes. They also have equal radii. Find the ratio of the height of the cone to its radius. Use : to indicate ratio.</p>	<p><math>\overline{LA}</math> and <math>\overline{LT}</math> are tangent to circle <math>T</math> at <math>A</math> and <math>K</math> respectively. <math>\overline{LA}</math> is perpendicular to <math>\overline{LK}</math> and <math>AL = 16</math>. Find the length of arc <math>KEA</math> in terms of <math>\pi</math>.</p> 	<p>In <math>\triangle ABC</math>, the measure of <math>\angle A</math> is <math>60^\circ</math>, the measure of <math>\angle B</math> is <math>45^\circ</math> and <math>AC = 8</math>. Find the length of <math>CB</math>.</p>	<p>In the cube shown, each edge has length 6. <math>AB, AC, BC</math> are face diagonals. Find the area of <math>\triangle ABC</math>.</p> 

**Answer List**

***	*** $x - y = 2$	*** $50^\circ$ ( $^\circ$ optional)
4-1. $\frac{3}{4} : 1$	4-2. $\frac{4000}{3}$ (or $1333.\bar{3}$ or $1333\frac{1}{3}$ )	4-5. $220^\circ$ ( $^\circ$ optional)
4-6. $0 < A \leq 50$	4-7. $2592 - 32\pi$	4-8. $\frac{48}{7}$ (or $6\frac{6}{7}$ )
4-9. $2m\sqrt{3}$	4-12. $5 : 7$	4-13. 81
4-14. 32.78	4-15. $120^\circ$ ( $^\circ$ optional)	4-16. (3, 4)
4-19. $\frac{1}{7}$	4-20. 31	4-21. 24
4-22. $1956$ (or $1.956 \times 10^3$ )	4-23. $6\pi + 4$ (or $2(3\pi + 2)$ )	4-26. $108\sqrt{3}$
4-27. $15^\circ$ ( $^\circ$ optional)	4-28. $\frac{648}{5}$ (or $129.6$ or $129\frac{3}{5}$ )	4-29. $\frac{1}{7}$ (or $0.\overline{142857}$ )
4-30. $3x - 5y = 31$	*** 4	*** 4 : 1
*** $24\pi$	*** $4\sqrt{6}$	*** $18\sqrt{3}$

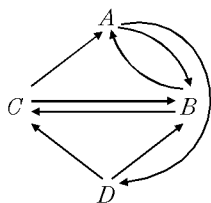
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**Catalog List**

*** ISC BE 20	*** ISC BB 66	*** ISC BE 41
4-1. ISC BD 75	4-2. ISC BA 76	4-5. ISC BB 31
4-6. ISC BF 58	4-7. ISC BG 39	4-8. ISC BC 32
4-9. ISC BF 21	4-12. ISC BE 9	4-13. ISC BG 72
4-14. ISC BD 51	4-15. ISC BD 44	4-16. ISC BC 18
4-19. ISC BF 63	4-20. ISC BG 12	4-21. ISC BC 73
4-22. ISC BA 49	4-23. ISC BA 12	4-26. ISC BG 24
4-27. ISC BD 14	4-28. ISC BB 52	4-29. ISC BE 80
4-30. ISC BB 2	*** ISC BD 24	*** ISC BF 8
*** ISC BG 57	*** ISC BA 34	*** ISC BC 48

## Team Competition Calculating

1. A teacher's 75 math students are to be lined up from left to right for a photograph. In how many different ways could this be done?
2. Given this communication network in which each arrow means a path of one step. By exactly how many paths of either 1, 2 or 3 steps can the person at  $C$  send a message to the person at  $B$ ? [Assume people don't send messages to themselves.]



3. A paper company plans to make boxes without tops from sheets of cardboard 12 inches wide and 16 inches long. They will cut out four squares of side  $x$  from the corners of the sheet and fold up the edges. What is the maximum volume, in cubic inches, of a box? Round your answer to the nearest hundredth.

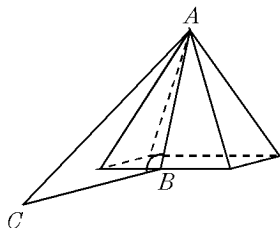
4. Find the largest value of  $x$  for which 
$$\begin{vmatrix} 3.1 & x & 2.7 \\ x & 5.1 & 4.9 \\ 8.3 & 4.2 & 6.2 \end{vmatrix} = \log_5 8.$$

5. Find the smallest prime  $p$  such that the sum of all the reciprocals of primes up to and including  $p$  exceeds 1.7.
6. Solve for  $x$ :  $9.5x^2 - 3.2x - 6.12 = 0$
7. Let  $a = 1.125$ . What is the smallest integer  $n$  such that the product  $a \cdot a^2 \cdot a^3 \cdots a^n$  is greater than 1,374,990? Do not use a decimal point in your answer.
8. A dart board is created in the shape of an isosceles right triangle. A circle is inscribed in the triangle. A dart is thrown at random but lands inside the triangle. Find the probability that the dart lands inside the circle. Give your answer as a decimal between 0 and 1.
9. Find the value of

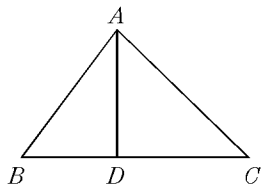
$$\frac{\tan 260^\circ + \tan 75^\circ}{1 - \tan 260^\circ \tan 75^\circ}$$

10. What is the remainder when  $4.72x^5 - 3.219x^4 + 7.4x - \pi$  is divided by  $x - e$ ?
11. A ski lift loading area is located 3500 ft on flat ground from the base of a hill whose top is a 1200 foot perpendicular distance above the flat ground. The angle of elevation of the cable from the loading area to the unloading area at the top of the hill is  $15^\circ$ . Assume the loading area, the unloading area, the base of the hill, and the foot of the perpendicular distance are coplaner. Rounded to the nearest minute, find the acute angle of elevation from the base of the hill to the top of the hill.

12. Find the sum of the entries in the  $9 \times 9$  matrix whose  $i, j$ -th entry is  $\log_{10} i^j$ .
13. Find the minimum distance from the point  $(8, 1)$  to the curve with equation  $\frac{x^2}{25} - \frac{y^2}{16} = 1$ . Give your answer rounded to 2 significant digits.
14. Find the sum of the coefficients of the polynomial  $\left(\frac{\sqrt{7}x}{3} - \frac{\pi y}{\sqrt{3}}\right)^8$ .
15. A cost-benefit curve to remove a certain pollutant from the air is given by  $y = \frac{3.84x}{96 - x}$ , where  $y$  is the cost, in thousands of dollars, of removing  $x$  percent of the pollutant, find the cost, in dollars, of removing 88.4% of the pollutant.
16. Arc  $AB$  has arclength 5 and is on a circle with equation  $x^2 + 2x - 8 = 8y - y^2$ . If the coordinates of point  $A$  are  $(-1, -1)$ , find the two possibilities for the  $x$ -coordinate of the other point.
17. The distance from point  $C$  to the base of the great pyramid of Khufu (Cheops) at Gizeh at point  $B$  is 312 ft. The angle of elevation,  $\angle ACB$ , to its top is  $25.5^\circ$ . If the pyramid makes an angle,  $\angle CBA$ , with the ground of  $141.8^\circ$ , find the slant height,  $AB$ , of the pyramid in feet to the nearest foot. Assume the ground is level.



18. In the diagram shown  $BD = 283$ ,  $DC = 192$ ,  $AC = 353$ , and  $AB = 400$ . Find  $AD$  to the nearest whole number.



19. In  $\triangle ABC$ , the altitude from  $C$  is 4.3. If  $\angle CAB = 37.58^\circ$  and  $AB = 11.8$ , what is the value of  $BC$ ?

20. Solve for  $x$ : 
$$\begin{vmatrix} 3 & 2 & 1 \\ 4 & 5 & x \\ -2 & 6 & x^2 \end{vmatrix} = 26$$

**Answer List**

- |                            |                    |                          |
|----------------------------|--------------------|--------------------------|
| 1. $2.481 \times 10^{109}$ | 2. 5               | 3. 194.07                |
| 4. 6.309                   | 5. 61              | 6. 0.9885, $-0.6517$     |
| 7. 15                      | 8. 0.5390          | 9. $-0.4663$             |
| 10. 541.7                  | 11. $50^\circ 48'$ | 12. 250.2                |
| 13. 2.6                    | 14. 0.5687         | 15. 44,670 (\$ optional) |
| 16. 3.207, $-5.207$        | 17. 611            | 18. 291                  |
| 19. 7.555                  | 20. 0.4197, 2.723  |                          |
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**Catalog List**

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|---------------|---------------|---------------|
| 1. ISC GB 29  | 2. ISC GF 55  | 3. ISC GG 28  |
| 4. ISC GE 26  | 5. ISC GC 43  | 6. ISC GC 22  |
| 7. ISC GA 74  | 8. ISC GB 71  | 9. ISC GD 23  |
| 10. ISC GF 37 | 11. ISC GG 74 | 12. ISC GE 62 |
| 13. ISC GA 40 | 14. ISC GA 4  | 15. ISC GB 18 |
| 16. ISC GE 46 | 17. ISC GG 7  | 18. ISC GF 19 |
| 19. ISC GD 53 | 20. ISC GC 66 |               |