Sample Documents

NC State Math Contest
(NCC)

EducAide Software

Math Team | Take-home Practice

Ms. Rogers | 1/15/99

1. Simplify: $\left(\dfrac{3x^{-3}}{5y^{-2}}\right)^{-2}$

a) $\dfrac{9x^6}{25y^4}$ b) $\dfrac{25y^4}{9x^6}$ c) $\dfrac{3x^5}{5y^4}$ d) $\dfrac{25x^6}{9y^4}$

e) none of these

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2. Let $(a, b)\,\Delta\, c = ac + bc - ab$ where a, b, c are rational numbers.

If $(a, b)\,\Delta\, x = x$. Solve for x in terms of a and b.

a) $\dfrac{a+b}{ab}$ b) $\dfrac{a+b-1}{ab}$ c) $\dfrac{ab}{a+b}$

d) $\dfrac{ab}{a+b-1}$ e) $\dfrac{ab}{1-a-b}$

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3. Upon calling the Carolina Twin Theatre, your receive the following recorded message:

> "Thank you for calling. Our features today are *Rocky XX* at the Twin 1 and *Rambo V* at the Twin 2. Each feature is run at two-hour intervals and, when you arrive, the odds will be 7 to 5 that *Rocky XX* will be showing next."

Eager to see the new Rocky picture, you arrive at the theatre at 1:20 and discover that *Rambo V* began at 1:10. Let T be the number of minutes remaining until *Rocky XX* starts. Then:

a) $T < 25$ b) $25 \le T < 40$

c) $40 \le T < 55$ d) $55 \le T < 70$

e) $70 \le T$

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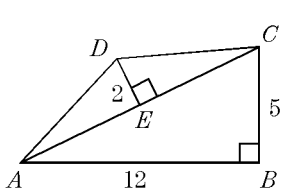
4. What is the area of $ABCD$?

a) 73 b) 43

c) 86 d) 56

e) 36



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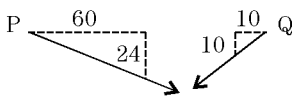
5. Two skin divers diving from boats 140 meters apart, represented by P and Q in the figure, plan to meet underwater between the two boats. One diver is 24 meters beneath the surface at a point 60 meters along the surface from his boat. The other diver is 10 meters beneath the surface at a point 10 meters along the surface from his boat. How far beneath the surface will the divers meet if they continue swimming at these same angles?



a) 30 m

b) 40 m

c) 50 m

d) 60 m

e) 100 m

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6. Let S be the set of all positive integers n such that

I. there exist distinct positive integers p and q with $\dfrac{1}{n} = \dfrac{1}{p} + \dfrac{1}{q}$

and

II. there exist distinct positive integers r and s with $\dfrac{1}{n} = \dfrac{1}{r} - \dfrac{1}{s}$.

Let T be the set of positive integers which are not in S. Then:

a) S is the empty set

b) S is nonempty but finite

c) S and T are both infinite

d) T is nonempty but finite

e) T is the empty set

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7. How many real numbers x satisfy $\Big|\Big|\big||x| - 2\big| - 4\Big| - 5\Big| = 1$?

a) 5 b) 6 c) 7 d) 8 e) 9

Math Team Take-home Practice Ms. Rogers 1/15/99

Answer List

1. d	2. d	3. d
4. b	5. b	6. d
7. b		

Catalog List

1. NCC AA 3	2. NCC CB 19	3. NCC CD 56
4. NCC EA 24	5. NCC EO 42	6. NCC FC 9
7. NCC BE 31		

1998 North Carolina
State Mathematics Contest

PART I: MULTIPLE CHOICE

1. How many different four digit integers can be formed by rearranging the digits of 1998?

   (a) 6 (b) 8 (c) 12 (d) 16 (e) 24

2. A and B are points on the circumference of a circle of radius r and center O. If the distance between A and B is r, what is the radian measure of $\angle AOB$?

   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) 1 (d) $\frac{\pi}{3}$ (e) $\frac{\pi}{2}$

3. When the North Carolina chapter of the "Spawn" fanclub quit accepting new members, it boasted a total of 500 devotees, 99% of which were male. One year later, no new members had joined the club. However, some males, but no females, had withdrawn from the club and the club was then only 96% male. How many members were then in this club?

   (a) fewer than 150
   (b) at least 150 but fewer than 250
   (c) at least 250 but fewer than 350
   (d) at least 350 but fewer than 450
   (e) at least 450

4. What is the units digit of the integer 2^{1998}?

   (a) 0 (b) 2 (c) 4 (d) 6 (e) 8

5. The members of a math club want to buy an engraved gold-plated calculator for their sponsor. If each member of the club contributes \$6.00 towards the purchase of the calculator, they will still be short \$16.00. On the other hand, if each members contributes \$9.00, they will have a surplus of \$11.00 after the purchase of the gift. What was the price of the gift in dollars?

   (a) \$61.00 (b) \$64.00 (c) \$70.00 (d) \$76.00 (e) \$79.00

6. PQR is a 3-4-5 right triangle with right angle at P. Points X on $\overline{PQ}$, Y on $\overline{QR}$ and Z on $\overline{PR}$ are chosen so that $PXYZ$ is a square. What is the length on the segment PX ?

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{5}{4}$ (d) $\frac{5}{3}$ (e) $\frac{12}{7}$

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7. The number of pairs of three digit numbers m and n such that $n = 3m$ and such that the union of the digits in the (base 10) expansions of m and n is the set $\{1, 2, 3, 6, 7, 8\}$ is

(a) 1 (b) 2 (c) 3 (d) 4

(e) more than 4

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8. There are 8 girls and 6 boys in the Math Club at Central High School. The Club needs to form a delegation to send to a conference, and the delegation must contain exactly two girls and two boys. The number of possible delegations that can be formed from the membership of the Club is

(a) 48 (b) 420 (c) 576 (d) 1680 (e) 2304

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9. Let $T = \{1, 2, 3, \ldots, 13\}$. A function $f : T \to T$ is said to be one-to-one if $t_1 \neq t_2$ inplies that $f(t_1) \neq f(t_2)$. The number of one-to-one functions f such that $t + f(t)$ is a perfect square, for every t in T, is

(a) 0 (b) 1 (c) 2 (d) 3

(e) more than 3

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10. The vertices of a square of side s, listed in clockwise order, are P, Q, R and S. M is the midpoint of $\overline{PQ}$ and N is the midpoint of $\overline{RS}$. Let V be the intersection of $\overline{SM}$ and $\overline{PN}$, and let W be the intersection of $\overline{RM}$ and $\overline{QN}$. Then the area of the quadrilateral $MWNV$ is

(a) $\frac{s^2}{12}$ (b) $\frac{s^2}{6}$ (c) $\frac{s^2}{4}$ (d) $\frac{s^2}{3}$ (e) $\frac{s^2}{2}$

1998 North Carolina State Mathematics Contest 9/24/98

Answer List

1. c	2. d	3. a
4. c	5. c	6. e
7. b	8. b	9. b
10. c		

Catalog List

1. NCC CD 104	2. NCC EH 38	3. NCC DB 56
4. NCC FB 32	5. NCC DC 96	6. NCC EA 54
7. NCC FC 16	8. NCC CD 105	9. NCC BH 74
10. NCC EC 91		