

Sample Documents

NSML Math Contest
(NSM)

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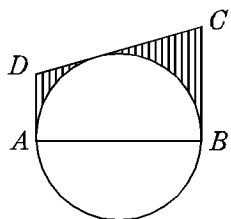
Submit your answers by the end of class Wednesday. For each correct answer, you will receive one-half extra credit point on next week's final.

1. A radiator is filled with water. One-fourth is drained and replaced with antifreeze. Later, $\frac{1}{4}$ of this mixture is drained and again replaced with antifreeze. The process is repeated a third time. After the three tasks, what fractional part of the radiator is filled with antifreeze?

2. Find the values of m for which the following polynomial will be factorable into the product of two linear factors with integral coefficients.

$$2x^2 + mxy - 6y^2 + 5x + 9y - 3$$

3. In the figure shown, \overline{AB} is a diameter and \overline{BC} , \overline{CD} , and \overline{AD} are each tangent to the circle. The radius of the circle is 4 and $CD = 12$. Find the area of the shaded region. Express your answer in terms of π .



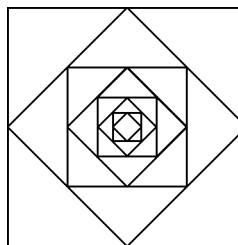
4. A small satellite is 250 km above a planet. A camera is on the satellite and the angle that it must turn from looking directly down onto the planet to looking at the horizon of the planet is 75° . To the nearest kilometer, what is the diameter of the planet?

5. Let $f(x) = \log\left(\frac{x-1}{x+1}\right)$, and $g(x) = \left(\frac{x^3+3x}{-3x^2-1}\right)$. Find $H(x) = f(g(x))$, and write H as a function of f .

6. Solve for x over the set of reals.

$$|x^2 + 3x - 4| + |x - 3| < 12$$

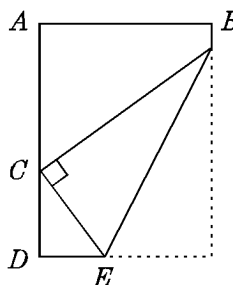
7. In the figure shown, the inscribed squares are formed by joining the midpoints of the sides of the circumscribed squares. The area of the largest square is 16. If it is assumed that this procedure is continued to a limiting point, then what is the sum of the perimeters of all the squares?



8. Two dice have faces which are numbered from -1 through 4, instead of the usual 1 through 6. When the two dice are rolled, what is the probability that the product of the two numbers is greater than the sum?

9. Determine: $\log_3 \sqrt[4]{729 \sqrt[3]{27^{\frac{2}{3}}}}$

10. In the figure shown, a rectangular sheet of paper is folded such that one corner touches the edge of the longer side. If $AD = 12$, $AB = 8$, and $\sin \theta = 0.4$, then what is the length of \overline{DC} ?



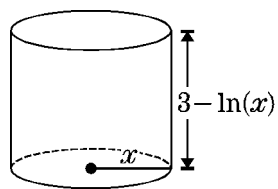
Answer List

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|------------------------------|--------------------------|--|
| 1. $\frac{37}{64}$ | 2. $-11, -4$ | 3. $48 - 8\pi$ |
| 4. 14,174 km | 5. $H(f) = -3f = -3f(x)$ | 6. $-1 - \sqrt{14} < x < -1 + \sqrt{14}$ |
| 7. $32 + 16\sqrt{2}$ | 8. $\frac{11}{36}$ | 9. $\frac{5}{3}$ |
| 10. $\frac{16\sqrt{21}}{21}$ | | |
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Catalog List

- | | | |
|---------------|--------------|--------------|
| 1. NSM AB 9 | 2. NSM AC 25 | 3. NSM BB 9 |
| 4. NSM CB 103 | 5. NSM AD 15 | 6. NSM GG 14 |
| 7. NSM GF 9 | 8. NSM GF 86 | 9. NSM EB 22 |
| 10. NSM GG 30 | | |

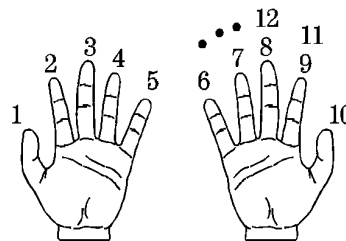
The formula for the volume of a cylinder is $V = \pi r^2 h$. The cylinder pictured has a radius of x units and a height of $3 - \ln(x)$ units. Find the maximum value, in cubic units, of such a cylinder. Round your answer to the nearest whole number.



“Two wrongs make a right,” at least in addition. Replace each of the letters in the following by digits 1 to 9 to make the sum correct.

$$\begin{array}{r} \text{WRONG} \\ + \text{WRONG} \\ \hline \text{RIGHT} \end{array}$$

A person counts on their fingers, as shown. Neither thumb receives two counts in a row. On what finger will the number 1990 be counted? Give your answer as a number from 1 to 10 which matches the finger in the drawing.



Solve the system for (x, y, z) :

$$\begin{aligned} x^2 + y^2 + z^2 &= 317 \\ x + y + z &= 25 \\ xy &= 10 \end{aligned}$$

Give the ordered triple having maximum x value as your answer.

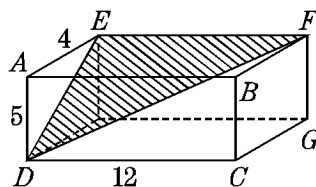
Given the following:

$$f \circ g(x) = 3x - 16$$

$$g \circ f(x) = 3x - 104$$

If f and g are both linear functions, and the graphs of f and g intersect at the point $(-4, -20)$, then find $f(x)$ and $g(x)$.

The rectangular solid shown, has a height of 5 ($AD = 5$), a length of 12 ($DC = 12$) and a depth of 4 ($AE = 4$). Find the area of $\triangle EDF$.



Let a be the number of solutions to $3 \sec x + 2 = 0$ in $[0, 2\pi]$.

Let b be the number of solutions to $2 \tan 3x = -1$ in $[0, 2\pi]$.

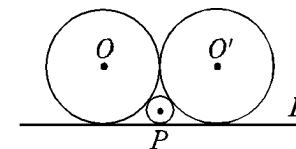
Let c be the number of solutions to $2 \cos 3\pi x + \sin \pi x = 0$ in $[0, 2\pi]$.

Let d be the number of solutions to $2 \cos^2 x = 1 + \cos 2x$ in $[0, 2\pi]$.

To the nearest integer, evaluate the expression

$$3^a + b^2 + \frac{c}{d}$$

Circles with centers O , O' , and P are each tangent to line L and also mutually tangent. If the radii of circle O and circle O' are equal, and the radius of circle P is 6, then what is the radius of the larger circles?



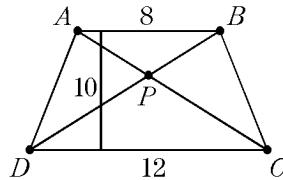
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1997-98 Sophomore Comp #3

1. A 400 meter track has two semicircular ends and two straight segments. The 400 meters represents the distance the runner on the innermost lane will run in a single lap. If each lane is one meter wide, for each 400 meters how much of a lead must the runner in the second lane be given so that each runs the same distance? Give your answer to the nearest hundredth of a meter (one centimeter is 0.01 meters). 1. _____

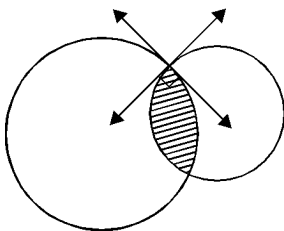


2. The diagonals of a rectangle are each 20 and the ratio of the length to the width is 5 : 4. What is the area of the rectangle? 2. _____

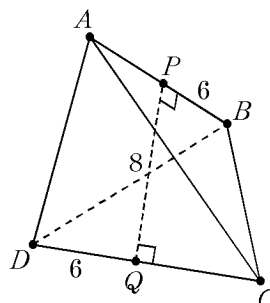
3. An isosceles trapezoid has parallel bases $AB = 8$ and $DC = 12$. The diagonals AC and BD intersect at the point P . If the height of the trapezoid is 10, then what is the area of $\triangle BPC$?



4. Two circles are “orthogonal” if their tangent lines at the point of intersection are perpendicular. If the areas of two orthogonal circles are 4π and 12π , then what is the area of the region for which the two circles intersect? 4. _____



5. In the tetrahedron shown, $AB = CD = 6$ are two skew edges. P and Q are the midpoints of \overline{AB} and \overline{CD} respectively, \overline{PQ} is perpendicular to each, and $PQ = 8$. What is the volume of the tetrahedron? 5. _____



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Answer List

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|----------------------------------|---------------------|-------|
| 1. 6.28 m | 2. $\frac{800}{41}$ | 3. 24 |
| 4. $\frac{10\pi}{3} - 4\sqrt{3}$ | 5. 48 | |
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Catalog List

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|--------------|--------------|--------------|
| 1. NSM BD 51 | 2. NSM BD 52 | 3. NSM BD 53 |
| 4. NSM BD 54 | 5. NSM BD 55 | |